Syllabus: Application of Biot-Savart's law to determine the magnetic field of a straight conductor, circular coil.

Magnetic field due to a straight conductor

Let a straight wire be considered whose magnetic field is to be determined at a certain point P which is nearby the conductor at a distance 'a'. Let a small portion of length $\vec{dl}$ be considered whose distance from P is '$\vec{r}$'.
From Biot-savart law, magnetic field due to current carrying element dl at point P is

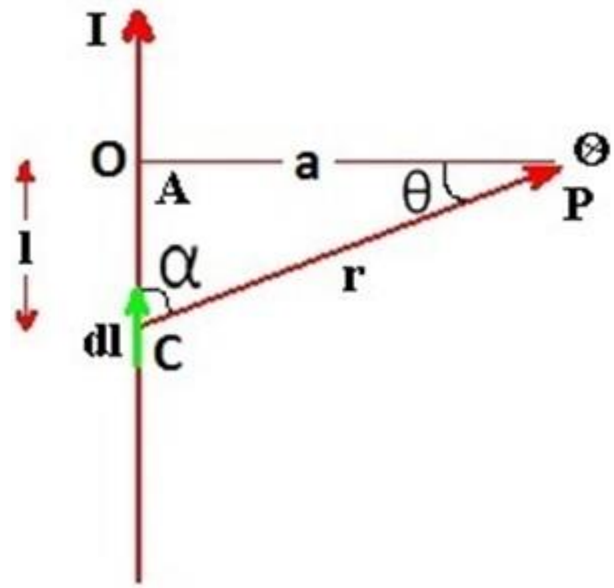
$$dB = \frac{\mu_\circ}{4\pi}\frac{Idlsin\alpha}{r^2} ---------(i)$$

$$from\ fig, sin\alpha = \frac{a}{r} = cos\theta$$

$$r = \frac{a}{cos\theta} ------(ii)$$

$$again, tan\theta = \frac{a}{l}$$

$$dl = a\sec^2\theta d\theta\ ------(iii)$$



From the above three equations

$$dB = \frac{\mu_\circ}{4\pi}\frac{Ia\sec^2\theta d\theta\ cos\theta}{(\frac{a}{cos\theta})^2}$$

$$dB = \frac{\mu_\circ}{4\pi}\frac{Ia\sec^2\theta d\theta\ cos\theta}{(a)^2}\cos^2\theta$$

$$dB = \frac{\mu_\circ}{4\pi}\frac{Icos\theta d\theta}{a}$$

Total magnetic field due to straight current carrying conductor is

$$B = \int_{-\theta_1}^{\theta_2} \frac{\mu_\circ}{4\pi} \frac{I cos\theta d\theta}{a}$$

$$B = \frac{\mu_\circ}{4\pi} \frac{I}{a} \int_{-\theta_1}^{\theta_2} cos\theta d\theta$$

$$B = \frac{\mu_\circ}{4\pi} \frac{I}{a} [sin\theta] \begin{matrix} \theta_2 \\ -\theta_1 \end{matrix}$$

$$B = \frac{\mu_\circ}{4\pi} \frac{I}{a} (sin\theta_2 + sin\theta_1)$$

This is the final expression for total magnetic field due to straight current carrying conductor.

If the conductor having infinite length then,

$$\theta_1 = \theta_2 = \frac{\pi}{2}$$

$$B = \frac{\mu_\circ}{4\pi} \frac{I}{a} (sin\frac{\pi}{2} + sin\frac{\pi}{2})$$

$$B = \frac{\mu_\circ}{4\pi} \frac{I}{a} 2$$

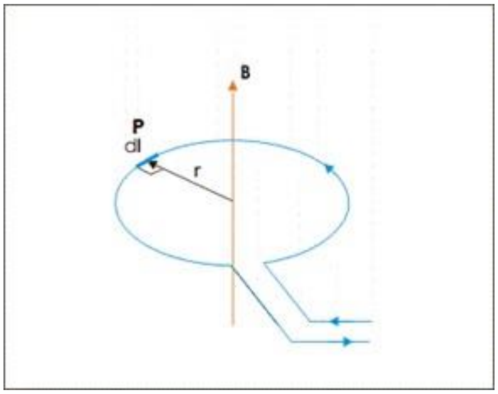
$$B = \frac{\mu_\circ}{2\pi} \frac{I}{a} \quad Tesla$$

Magnetic field due to current carrying circular coil at its center

Consider a circular current carrying coil having radius r and center O. When the current is passing through the circular coil, magnetic field is produced. To find the magnetic field at the center of the circular coil, consider a length of element dl at point p which is tangent to the circular coil. The angle between element dl and radius r is 90°.

According to the Biot-Savart law, the magnetic field at the center of the circular coil due to element dl is

$$dB = \frac{\mu_\circ}{4\pi}\frac{Idlsin\theta}{r^2} = \frac{\mu_\circ}{4\pi}\frac{Idlsin90}{r^2} = \frac{\mu_\circ}{4\pi}\frac{Idl}{r^2}$$



Total magnetic field due to the circular coil is

$$B = \int_0^{2\pi r} dB = \int_0^{2\pi r} \frac{\mu_\circ}{4\pi}\frac{Idl}{r^2} = \frac{\mu_\circ}{4\pi}\frac{I}{r^2}\int_0^{2\pi r} dl$$

$$= \frac{\mu_\circ}{4\pi}\frac{I}{r^2}[l]_0^{2\pi r}$$

$$= \frac{\mu_\circ}{4\pi}\frac{I}{r^2}2\pi r$$

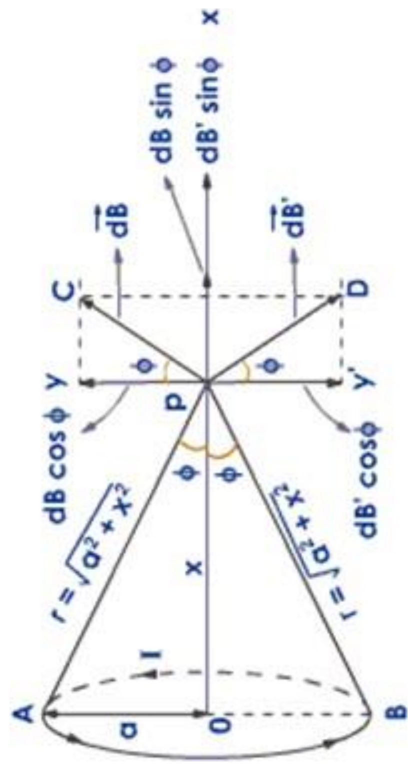
$$= \frac{\mu_\circ}{2}\frac{I}{r}$$

Magnetic field at the axis of the circular current carrying coil

Consider a circular coil having radius a and centre O from which current I flows in anticlockwise direction. The coil is placed at yz plane so that the centre of the coil coincide along x-axis. P be the any point at a distance x from the centre of the coil where we have to calculate the magnetic field. let dl be the small current carrying element at any point A at a distance r from the point P where $r = \sqrt{(x^2 + a^2)}$

the angle between r and dl is 90°. Then from Biot-Savart law, the magnetic field due to current carrying element dl is

$$dB = \frac{\mu_\circ}{4\pi}\frac{Idlsin\theta}{r^2} = \frac{\mu_\circ}{4\pi}\frac{Idlsin90}{r^2} = \frac{\mu_\circ}{4\pi}\frac{Idl}{r^2}$$



the direction of magnetic field is perpendicular to the plane containing dl and r. So the magnetic field dB has two components

$dBcos/$ is along the y-axis

$dBs0n/$ is along the x-axis

Similarly, consider another current carrying element dl' which is diametrically opposite to the point A. The magnetic field due to this current carrying element dB′ also has two components

$dB'cos/$ is along the y-axis

$dB's0n/$ is along the x-axis

Here both $dBcos/$ and $dB'cos/$ are equal in magnitude and opposite in direction. So they cancle each other. Similarly, the components $dBs0n/$ and $dB's0n/$ are equal in magnitude and in same direction so they adds up.

Total magnetic field due to the circular current carrying coil at the axis is

$$B = \int_0^{2\pi a} dB sin\theta = \int_0^{2\pi a} \frac{\mu_\circ}{4\pi} \frac{Idl}{r^2} \frac{a}{r}$$

$$since\ sin\theta = \frac{a}{r} B = \int_0^{2\pi a} \frac{\mu_\circ}{4\pi} \frac{Idl}{(x^2+a^2)} \frac{a}{(x^2+a^2)^{\frac{1}{2}}} = \frac{\mu_\circ}{4\pi} \frac{Ia}{(x^2+a^2)^{\frac{3}{2}}} \int_0^{2\pi a} dl$$

$$B = \frac{\mu_\circ}{4\pi} \frac{Ia}{(x^2+a^2)^{\frac{3}{2}}} 2\pi a$$

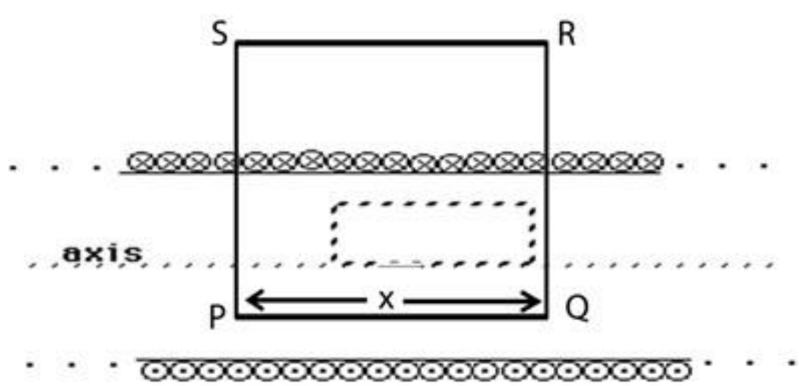
$$B = \frac{\mu_\circ}{2} \frac{Ia^2}{(x^2+a^2)^{\frac{3}{2}}}\ Tesla$$

This is the expression for magnetic field due to circular current carrying coil along its axis.

If the coil having N number of turns then magnetic field along its axis is

$$B = \frac{\mu_\circ}{2} \frac{INa^2}{(x^2+a^2)^{\frac{3}{2}}}\ Tesla$$

Magnetic field along axis of solenoid



A solenoid is a ling cylindrical coil having number of circular turns. Consider a solenoid having radius R consists of n number of turns per unit length. Let P be the point at a distance $x_\circ$ from the origin of the solenoid where we have to calculate the magnitude of the magnetic field. The current carrying element dx at a distance x from origin and a distance r from point P

$$r = \sqrt{(R^2 + (x_4 - x)^2)}$$

The magnetic field due to current carrying circular coil at any axis is

$$dB = \frac{\mu_\circ}{2}\frac{IR^2}{r^3} \times N$$

$$where\ N = ndx$$

$$then$$

$$dB = \frac{\mu_\circ}{2}\frac{nIR^2dx}{r^3} - - - - - - - - - - - (i)$$

$$sin\Phi = \frac{R}{r}$$

$$r = Rcosec\Phi - - - - - - - - - - - (a)$$

$$tan\Phi = \frac{R}{x_\circ - x}$$

$$x_\circ - x = Rcot\Phi$$

$$\frac{dx}{d\Phi} = Rcosec^2\Phi$$

$$dx = Rcosec^2\Phi d\Phi - - - - - - - - (b)$$

Now from above three equations, we get,

$$dB = \frac{\mu_\circ}{2}\frac{nIR^2Rcosec^2\Phi d\Phi}{R^3cosec^3\Phi}$$

$$dB = \frac{\mu_\circ}{2}nIsin\Phi d\Phi$$

Now total magnetic field can be obtained by integrating from Φ_1 to Φ_2, we get

$$B = \frac{\mu_\circ nI}{2}\int_{\Phi 1}^{\Phi 2} sin\Phi d\Phi$$

$$B = \frac{\mu_\circ nI}{2}[-cos\Phi]\begin{matrix}\Phi_2\\ \Phi_1\end{matrix}$$

$$B = \frac{\mu_\circ nI}{2}(cos\Phi_1 - cos\Phi_2)$$

Hence this expression gives the magnetic field at point p of the solenoid of finite length. For infinite long solenoid $6_7 = 0, 6_7 = \pi$

So

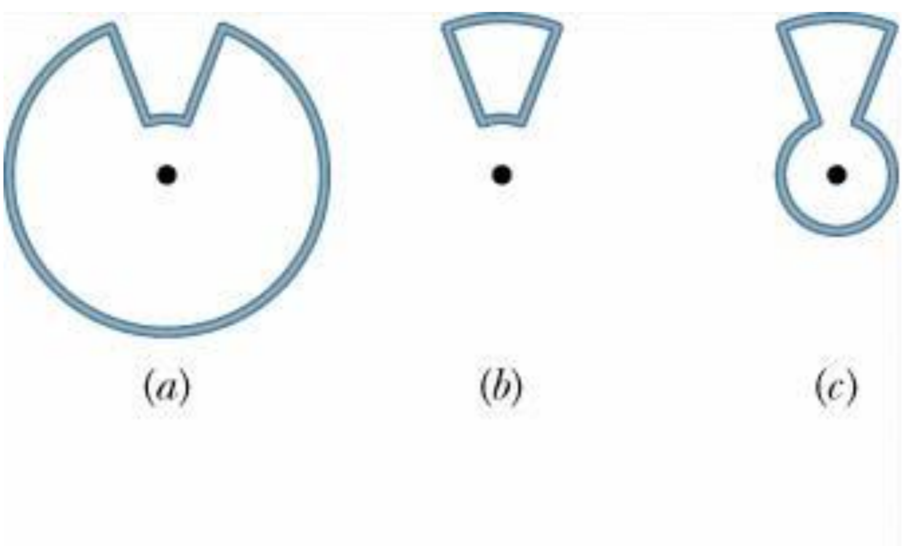
$$B = \frac{\mu_\circ nI}{2}(cos0 - cos\pi)$$

$$B = \frac{\mu_\circ nI}{2}(1+1)$$

$$B = \mu_\circ nI\ Tesla$$

Home work:

Find the magnetic field for the following partial loops



Note on problems when you have to evaluate a B field at a point from several partial loops

Only loop parts contribute, proportional to angle (previous slide)

Straight sections aimed at point contribute exactly 0

Be careful about signs, e.g. in (b) fields partially cancel, whereas in (a) and (c) they add