

Standard Deviation

Standard deviation: Standard deviation is the measure of how spread out or dispersed the value in a data set are.

Definition: Standard deviation (denoted as σ or population for sample) shows how much the values deviates from the mean (average of the data set).

Formula: For a population

$$\sigma = \sqrt{rac{1}{N}\sum_{i=1}^{N}(x_i-\mu)^2}$$

For sample:

$$s=\sqrt{rac{1}{n-1}\sum_{i=1}^n(x_i-ar{x})^2}$$

Note: We divide by n-1 (not n)—this is called **Bessel's correction**, and it corrects bias in the estimation.

Where:

- σ = population standard deviation
- s = sample standard deviation
- \sim N = number of values in the population
- $x \neq$ each value in the population
- μ = population mean
- n = number of values in the sample
- $ar{x}$ = sample mean



Key points:

- i. Low standard deviation: Data points are closed to the mean (less spread).
- ii. High standard deviation: Data points are most spread out from the mean.
- iii. It is always non-negative.
- iv. Unit of the standard deviation is the same as data.

Population vs Sample: Key Differences

Definition	Entire group of individuals or data points	Subset of the population
Size	Denoted by N (usually larger)	Denoted by n (usually smaller)
Mean	Population mean $\mu=rac{1}{N}\sum x_i$	Sample mean $ar{x} = rac{1}{n} \sum x_i$
Standard Deviation	$\sigma = \sqrt{rac{1}{N}\sum (x_i - \mu)^2}$	$s = \sqrt{rac{1}{n-1}\sum (x_i - ar{x})^2}$
Denominator in SD	Divide by N	Divide by n - 1 (Bessel's correction)
Purpose	Describes the actual data spread in a full set	Estimates the spread of the entire population based on a sample
Accuracy	Precise (if all data is known)	Approximate (used for inference)
Use Case	When data for the entire population is available	When data is available only for a part of the population

Q1. A student performs an acid-base titration 5 times to determine the volume of NaOH (titrate) requires to neutralized a fixed volume of HCl(analyte). The volume reading (ml) are:

25.1, 25.0, 24.9, 25.2, 25.0

Find the standard deviation.



.....

Solution:

Step1: Calculate the mean (or average volume)

$$\bar{x} = \frac{25.1 + 25.0 + 24.9 + 25.2 + 25.0}{5} \ ml$$

$$=\frac{125.2}{5}$$
 ml = 25.04 ml

Step 2: Find the deviation from the mean and their squares

Volume (ml)	Deviation $(x_i - \bar{x})$	Square deviation
		$(x_i - \bar{x})^2$
25.1	(25.1-25.04)=0.06	$(0.06)^2 = 0.0036$
25.0	(25.0-25.04) = -0.04	$(-0.04)^2 = 0.016$
24.9	(24.9-25.04) = -0.14	$(-0.14)^2 = 0.0196$
25.2	(25.2-25.04)=0.16	$(0.16)^2 = 0.0256$
25.0	(25.0-25.04)=- 0.04	$(-0.04)^2 = 0.0016$

$$s=\sqrt{rac{1}{n-1}\sum_{i=1}^n(x_i-ar{x})^2}$$

$$S = \sqrt{\frac{0.0036 + 0.0016 + 0.0196 + 0.0256 + 0.0016}{5 - 1}}$$

$$= \sqrt{\frac{0.052}{4}} = \sqrt{0.013} = 0.114 \text{ m}$$

Interpretation:

Mean value used =25.04 ml

Standard deviation =0.114ml



Small standard deviation indicates the titration result is precise, meaning the student perform the titration consistently.

Mean, Median and Mode

1. Mean (average): Mean is the sum of all values divided by number of values.

Formula: Mean=Sum of all values number of values

Example: Data: 2,4,6,8,10

$$Mean = \frac{2+4+6+8+10}{5} = \frac{30}{5} = 6$$

2. Median (Middle value): The median is the middle value when the data values are arranged in ascending order.

Steps: i. Arrange data in order.

- ii. If odd number of values, middle value is the median.
- iii. If <u>even number</u>, average of the two-middle value is the median.

Example: In odd case: data: 3,5,7 Median =5

In even case: data 4,6,8,10, Median =
$$\frac{6+8}{2}$$
 = 7

3. Mode (most frequent value): The mode is the number that appears most frequently.

Example: data: 2, 3, 3, 5,7

Mode= 3 (Because it occurs repeatedly).

N.B.: No mode if all numbers occur once.

('bimodal' or multimodal)More than one mode if two or more value occurs equally.