How to convert rectangular coordinates to polar coordinates, and vice-versa.

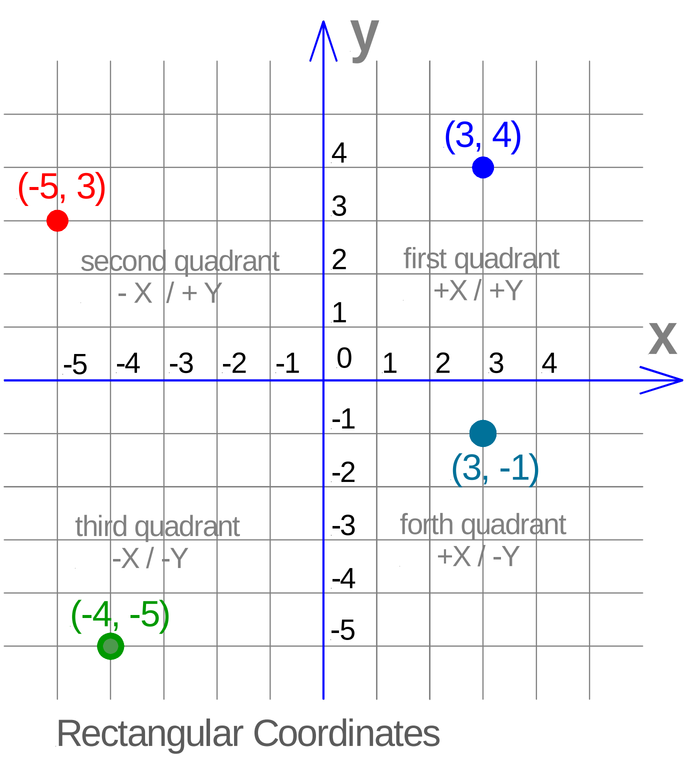
Rectangular coordinates

Rectangular coordinates and polar coordinates are two different ways of using two numbers to locate a point on a plane.

Rectangular coordinates are in the form (x,y), where 'x' and 'y' are the horizontal and vertical distances from the origin. In other words the first figure refers always to the distance from the origin in direction of the X-axis to the point and the second figure from the origin in the direction of the Y-axis to the point

In the diagram below the coordinates are:

(x,y) in the **1st** quadrant are	(3, 4)
(x,y) in the **2nd** quadrant	(–5, 3)
(x,y) in the **3rd** quadrant	(–4, –5) and
(x,y) in the **4th** quadrant	(3, –1).



The axes of a two-dimensional Cartesian system divide a plane into four regions, called **quadrants**, each bounded by two half-axes. These are often numbered from 1st to 4th and where the signs of the two coordinates are

in the **1st** quadrant	**(+,+)**,
in the **2nd** quadrant	**(–,+)**,
in the **3rd** quadrant	**(–,–)**, and
in the **4th** quadrant	**(+,–)**.

As can be seen a **rectangular coordinate system** specifies each point in a plane by a pair of numerical **coordinates**, which are the distances from the point to two fixed perpendicular directed lines, measured in the same unit of length.

Convertion of rectangular to polar coordinates

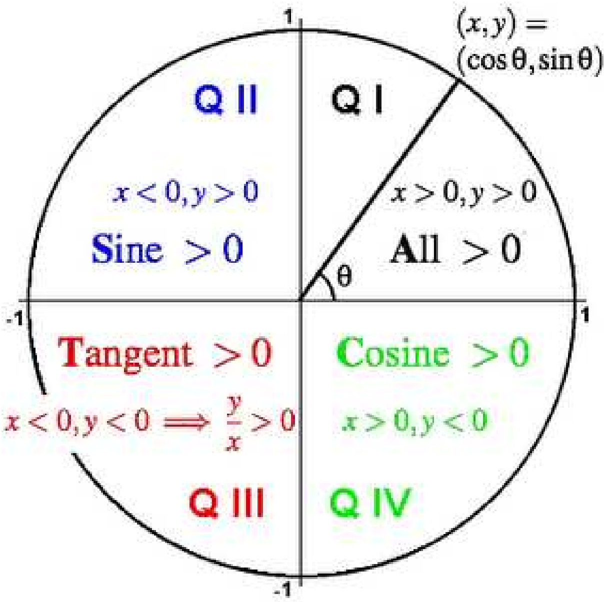
The rectangular coordinates (x,y) and polar coordinates (r, θ) are related as follows:

$x = r \cos \theta$ and $y = r \sin \theta$

$R^2 = x^2 + y^2$ and $\tan \theta = y/x$

Signs of sine, cosine and tangent, by Quadrant

The definition of the trigonometric functions cosine and sine in terms the coordinates of points lying on the unit circle tell us the signs of the trigonometric functions in each of the four quadrants, based on the signs of the x and y coordinates in each quadrant.



First Quadrant
For an angle in the first quadrant the point P has positive x and y coordinates. Therefore: In Quadrant I, cos(θ) > 0, sin(θ) > 0 and tan(θ) > 0 (All positive).

2nd Quadrant
For an angle in the second quadrant the point P has negative x coordinate and positive y coordinate. Therefore: In Quadrant II, cos(θ) < 0, sin(θ) > 0 and tan(θ) < 0 (Sine positive).

3rd Quadrant
For an angle in the third quadrant the point P has negative x and y coordinates. Therefore: In Quadrant III, cos(θ) < 0, sin(θ) < 0 and tan(θ) > 0 (Tangent positive).

4th Quadrant
For an angle in the fourth quadrant the point P has positive x coordinate and negative y coordinate. Therefore: In Quadrant IV, cos(θ) > 0, sin(θ) < 0 and tan(θ) < 0 (Cosine positive).

The quadrants in which cosine, sine and tangent are positive are often remembered using a favorite mnemonic.
One example: **All Students Take Calculus**.

Reference:
http://sriamanmathblog.blogspot.com/2009/09/signs-of-sine-cosine-and-tangent-by.html

Example:

(a) A point has rectangular coordinates: (3, 4) as shown in the rectangular coordinates. Convert this rectangular coordinates to polar coordinates.

Solution: r = square root of $(3^2 + 4^2) = 5$, $\theta = \tan^{-1}(4/3) = 53.13^o$

so (r, θ) = (5, 53.13º)

(b) A point has rectangular coordinates: (**— 5, 3**) (point is in 2nd quadrant)

Solution: R = square root of $[(-5)^2 + 3^2] = 5.83$, $\theta = \tan^{-1}(3/5) = 180° - 30.964°$

so (r, θ) = [5.83, (180° — 30.964°)]
(5.83, 149.04°)

(c) A point has rectangular coordinates: (**— 4, —5**) (point is in 3rd quadrant)

Solution: R = square root of $[(-4)^2 + (-5)^2] = 6.4$, $\theta = \tan^{-1}(5/4) = 180° - 51.34°$

so (r, θ) = [5.83, (180°+ 51.34°)]
(5.83, 231.34°)

(d) A point has rectangular coordinates: (3^2**, —1**) (point is in 4th quadrant)

Solution: R = square root of $[3^2 + (-1)^2] = 3.16$ $\theta = \tan^{-1}(-1/3) = 18.43°$

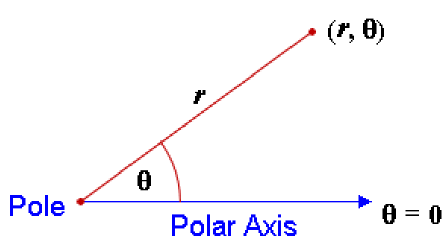
so (r, θ) = [5.83, (360° — 18.43°)]
(5.83, 378.43°)

The determination of θ is as follow:

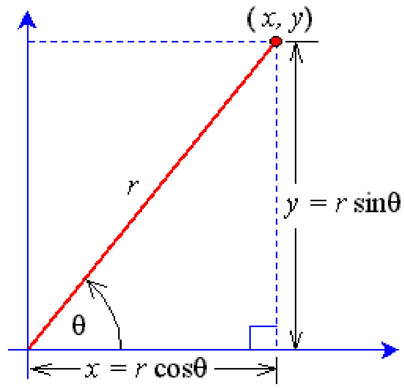
in the **1st** quadrant	**(+,+)**,	it is **θ**
in the **2nd** quadrant	**(−,+)**,	it is 180 **— θ**
in the **3rd** quadrant	**(−,−)**,	it is 180 **+ θ**
in the **4th** quadrant	**(+,−)**	it is 360 **— θ**

Polar Coordinates

In Surveying 2 the rectangular coordinate system is seldom use, but familiarity with the **polar coordinate system** is essential for the practical exercises.



Polar coordinates are in the form: (**r**, **θ**), where 'r' is the distance from the origin to the point, and '**θ**' is the angle measured from the positive 'x' axis to the point, as shown in the opposite diagram.



To convert between polar and rectangular coordinates, we make a right triangle to the point (x,y), like this:

Polar to Rectangular

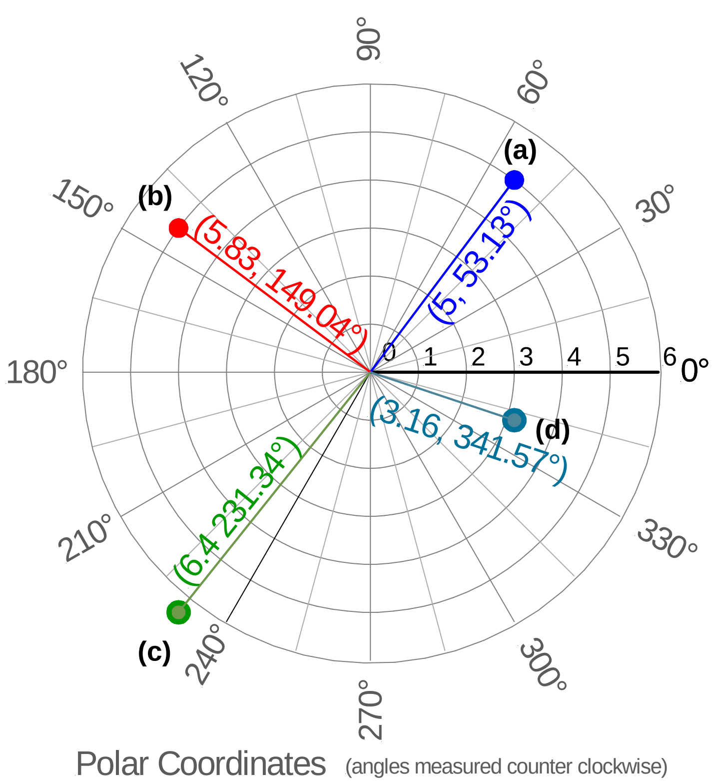
From the diagram above, these formulas convert **polar** coordinates **to rectangular** coordinates:

$x = r \times \cos\theta, \quad y = r \times \sin\theta$

So the **polar** point: (r,θ) can be converted **to rectangular** coordinates like this:

$$(r \times \cos\theta,\ r \times \sin\theta) \Rightarrow (x, y)$$

Polar coordinate system



A **circular coordinate system**, is a two-dimensional polar coordinate system, defined by an origin, *O*, and a fixed line (right half of the positive x-axis) leading from this pointdistance r from pole O to point P, and measure the angle theta between the axis and OP in a counterclockwise direction This line is also called polar axis.

The location of a point is determined by its distance from a fixed point (pole) at the centre of the coordinate space.

The distance r from pole O to point P (pole ray) and the angleθ (theta) between the axis and OP is measure in a anticlockwise direction

The polar coordinates in the opposite diagram are equial to the one of the rectangular coordinates shown on page 1

$$0 \leq \theta < 360°$$

Example:

(a) A point has polar coordinates: (5, 53.13°) as s hown in the above diagram.

Convert this polar coordinates to rectangular coordinates.

Solution: (x,y) = (5 x cos 53.13º, 5 x sin 53.13º) = (3, 4)

(b) A point has polar coordinates: (5.83, 149.04°) as shown in the above diagram.

Convert this polar coordinates to rectangular coordinates.

Solution: (x,y) = (5.83 x cos 149.04°, 5.83 x sin 149.04°) = (–5, 3)

(c) A point has polar coordinates: (6.4, 218.66°) a s shown in the above diagram.

Convert this polar coordinates to rectangular coordinates.

Solution: (x,y) = (6.4 x cos 231.34°, 6.4 x sin 231.34°) = (–4, –5)

(a) A point has polar coordinates: (3.16, 341.57°) as shown in the above diagram.

Convert this polar coordinates to rectangular coordinates.

Solution: (x,y) = (3.16 x cos 341.57º, 3.16 x sin 341.57º) = (3, –1)

I you like to the experiment with rectangular or cartesian and polar coordinate systems access the web page below.

There are three applets (1) rectangular (Cartesian) coordinates, (2) Coordinate systems and (3) polar coordinates that's great fun to play with.

http://www.univie.ac.at/future.media/moe/galerie/zeich/zeich.html