Viscosity

6. Critical Velocity

During the flow of a liquid through a tube below a certain velocity the flow remains streamline but above that velocity it becomes turbulent. This critical velocity is determined by (i) the radius of the tube, (ii) the density of the liquid and (iii) coefficient of viscosity.

The critical velocity is related to the above factors by the equation.

$v_C=k\eta/\rho r$,

where η= viscosity coefficient of the liquid, k is a dimensionless parameter called Raynolds number, ρ =density of the liquid and r= radius of the capillary.

When k<1000 for a narrow tube the flow remains laminar. If diameter of the tube is considered instead of radius, Raynolds' number for laminar flow will be 2000 and when the number exceeds 3000, the flow becomes turbulent. Between 2000-3000, the flow changes from one type to another.

7. Viscosity of gas:

This property is exhibited by both gases and liquids and is a measure of fractional resistance that a fluid in motion offers to an applied shearing force. In gases, viscosity is due to the exchange of molecules from one layer to other. As a result of continuous exchange of molecules, there is a transfer of momentum of molecules from one layer to the other and consequently, their velocities. The exchange of molecules from the faster moving layer to the slower moving layer results in a decrease in the speed of the molecule in the faster layer and an increase in the speed of the molecules in the slower moving layer. The net result of this exchange of molecules is a tendency towards equalizing the flow rate of the different parts of the gas.

To define viscosity, let us consider a gas flowing in parallel planes along the x- direction with a velocity v **(Fig 4) .** These layers are separated by a distance λ from each other. The layer adjacent to the x-axis has a velocity $v\approx 0$ and it increases with increasing distance along y-axis. If the velocity gradient is given dv/dy along y- axis; then if the gradient is uniform the average velocity of flow at any distance λ above a layer AA' is given by $v+\lambda(dv/dy)$. Similarly the average velocity of flow at any distance λ below it is equal to $v-\lambda(dv/dy)$ as shown in the figure. The flow of gas can be understood in terms of a force required to move a layer of gas relative to another layer. The force F required to maintain a steady velocity difference dv between any two parallel layers is directly proportional to the area A of the layers and the velocity gradient, dv/dy. Consequently

$$F= -\eta A\,(dv/dz) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (3)$$

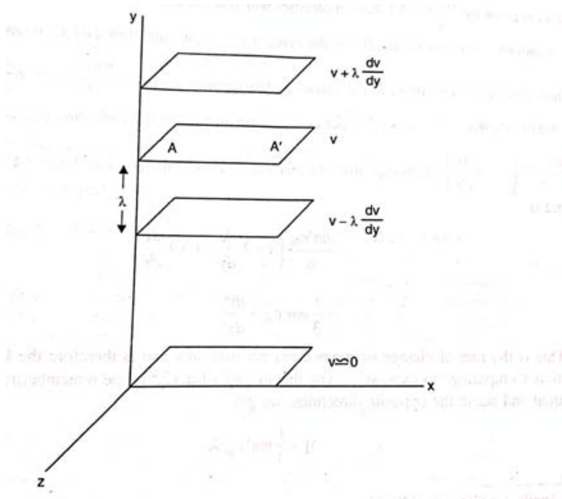

Figure 4: Stream line flow in gases due to a shearing action

Where η is the constant of proportionality and is called the coefficient of viscosity.

In order to derive an expression for the coefficient of viscosity, let us assume that the number of molecules of a gas per unit volume is n' and the average velocity of gas molecule is c_{av}. Since the motion of the molecule is completely random, it may be assume that n'/3 molecules are moving alone each axis. Of these n'/3 molecules, half (n'/6) would be moving in one direction (say along y-axis) and n'/6 in the opposite direction (-y axis). Referring to Fig 4 and remembering that viscosity in gases arises due to transfer of momentum from one layer to another, we calculate the net momentum transferred per unit area per second to a layer (say AA') by the molecules from adjacent layer separated by a distance λ. The number of molecules moving upward (or downward) through unit area in one second is given by $n'C_{av}/6$. All those molecules which are at a distance λ from AA' will strike this layer and transfer to it their momenta . If m is the mass of a gas molecule, then the total momentum transferred per unit area per second by molecules moving downward is given by $mn' C_{av}/6 (v+ \lambda dv/dz)$. Similarly the momentum transferred to layer AA' per unit area per second by the molecules moving upward is equal to $mn' C_{av}/6 (v-\lambda dv/dz)$. Consequently the net momentum transferred to layer AA' per unit area per second is

$$mn' C_{av}/6\,(v+ \lambda dv/dz- v+ \lambda dv/dz)$$
$$=1/3\ mn' C_{av}\lambda dv/dz$$

This is the rate of change of momentum per unit area and is therefore the force acting along x direction. Comparing this expression with that given by the eq. 3 and remembering that the two forces are equal and act in the opposite directions , we get

$$\eta=1/3mn'C_{av}\lambda \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

However, a more rigorous derivation is obtained by taking into account the distribution of molecular speeds. The resulting expression is

$$\eta=1/2mn'C_{av}\lambda \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Substituting the value of C_{av} in equation 5 , we get

$$\eta = \frac{mn'\lambda}{2}\sqrt{\frac{8kT}{\pi m}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(6)}$$

It is clear from eq. 6 that the viscosity of a gas is proportional to the square root of the absolute temp. In actual practice, it is found that it increases more rapidly that implied by this relationship. The increase in η with temperature is due to the fact that the momentum is transferred more rapidly through a given area and a greater force has to be applied to maintain the motion of the layer of the gas.

8. Viscosity of liquid:

The effect of temperature on the viscosity of a liquid is strikingly different from that of a gas. The coefficient of viscosity of gases increases with the increase of temperature, while those of liquids decreases due to weakening of intermolecular attractions. Various empirical equation relating viscosity with temperature have been proposed, but the expression given by Arrhenus and Guzman is the most satisfactory and is given by

$$\eta = Ae^{E_a/RT}$$

$$\ln\eta = \ln A + E_a/RT$$

$$\log\eta = \text{constant} + E_a/2.303RT \qquad \ldots\ldots\ldots\ldots\ldots \text{(7)}$$

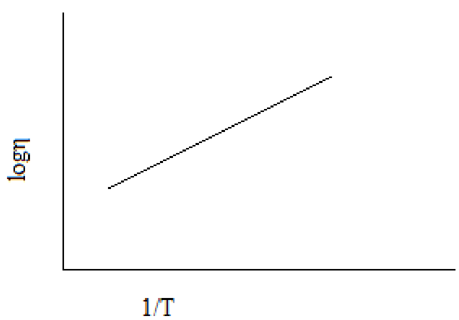

Fig 5: Plot of logη vs 1/T

Where A is a constant and E_a is the activation energy for the viscous flow and R is the gas constant and T the temperature in degree absolute. It follows from eq. 7 that plot of logη vs 1/T will be straight line with slope $E_a/2.303R$, from which E_a can be calculated.

9. Measurement of Viscosity

The rate of laminar flow of a liquid through a capillary tube at a constant pressure is related to the viscosity of the liquid. The relationship was first derived by Poiseuille in 1844 and is given by the equation

$$\eta = \frac{\pi P r^4 t}{8lV} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Where V is the volume of liquid of viscosity η which flows in time t through a capillary of radius r and length l under a deriving pressure P. Since P=hdg, where h is the height of the liquid column, d the density of the liquid and g the acceleration due to gravity. Substituting these values in eq. 8, we get

$$\eta = \frac{\pi hdgr^4 t}{8lV} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

If equal volume of two sliquids are allowed to flow through the same capillary under similar condition, then we have

$$\frac{\eta_1}{\eta_2} = \frac{\pi r^4 t_1 h d_1 g}{8lV} \times \frac{8lV}{\pi r^4 t_2 h d_2 g}$$

$$\frac{\eta_1}{\eta_2} = \frac{t_1 d_1}{t_2 d_2} \quad \ldots\ldots\ldots\ldots\ldots (10)$$

If d_1, d_2 and η_1, η_2 are known, determination of t_1 and t_2 enables us to calculate η_1, the coefficient of viscosity of the liquid under investigation.

Ostwald Viscometer Method: As mention above, the quantities t_1 and t_2 are most conveniently measured with an Ostwald viscometer shown in fig.

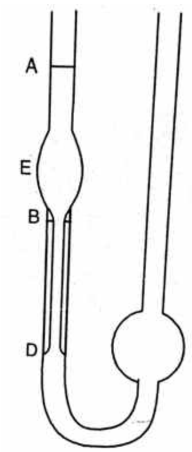

Fig 6. Ostwald Viscometer

It consists of a capillary tube BD through which a definite volume of liquid (between the marks A and B) is allowed to flow under the force of its own weight. To start with, definite quantity of a liquid is introduced into the viscometer and is then drawn up by suction into the bulb E until the liquid level is above the mark. The liquid is then allowed to drain, and the time necessary for the liquid level to fall from A to B is measured with the whole operation is repeated with the reference liquid, usually water. Knowing the times of flow for both the liquids, viscosity of the unknown liquid can be calculated using eq.10.

Falling Sphere Viscometer:

The motion of a body moving through a viscous medium is opposed by the fractional resistance of the medium. In order to maintain a uniform velocity, a driving force has to be applied to overcome the fractional force. The magnitude of the fractional force (F_r) depends on the velocity of the body and is given by the Stokes relation

$$F_r = 6\pi r\eta u$$

Where r is the radius of the spherical body and u its velocity through the medium having coefficient of viscosity η.

If d is the density of spherical body and d_0 the density of the medium, then the gravitational force, F_g

$$F_g = 4/3\pi r^3(d-d_0)g$$

When the rate of setting of the sphere in the liquid is constant, then the gravitational and frictional force are equal, i.e.,

$$6\pi r\eta u = 4/3\pi r^3(d-d_0)g$$

$$\eta = 2r^2(d-d_0)g/9u$$

This equation is known as Stokes' law and is applicable to the all of spherical bodies in fluids. By measuring the velocity u of the spherical body of known r and d, through a vertical column of liquid of density d_0, η may be calculated. This method is generally used for solution of high viscosities.

If the same spherical body is allowed to fall through the same distance in two liquids then the ratio of the viscosities is given by

$$\frac{\eta_1}{\eta_2} = \frac{(d-d'_0)}{(d-d''_0)} \frac{t_1}{t_2}$$

Where d'_0 and d''_0 are the densities of the medium and t_1 and t_2 are the times taken by the spherical body to fall through the same distance in the two media.