DUMKAL COLLEGE

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Department of Mathematics

Class Note

B.Sc. Mathematics (Major)
Semester VI
Course Code: MATH-M-T-09
Course Title: Linear Programming Problems & Game Theory

Unit 1: Introduction to Linear Programming Problems

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CLASS NOTE – Concept 1

(Mathematical Formulation of LPP)

What is Linear Programming?

Linear Programming is a mathematical technique used to find the best possible outcome (maximum profit or minimum cost) when resources are limited. The word **linear** means all equations and inequalities are straight-line relationships (no x^2, no xy, no $\sqrt{x}$). The word **programming** means planning — not computer coding.

> **Key idea:** We want to maximize or minimize a linear function subject to linear constraints.

Four Main Components of any LPP

Every LPP has exactly four parts. I will write them in a table as I remember:

Component	What it means	Example
Decision variables	What we decide (unknowns)	$x =$ chairs, $y =$ tables
Objective function	Goal (max or min)	$Z = 50x + 80y$ (profit)
Constraints	Limitations (inequalities)	$2x + 3y \leq 100$
Non-negativity	No negative amounts	$x \geq 0,\ y \geq 0$

Mathematical Form (Standard Form)

For a **maximization** problem with n variables and m constraints:

$$\begin{aligned}
&\text{Maximize } Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n \\
&\quad \text{subject to} \\
&\qquad a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq b_1 \\
&\qquad a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \leq b_2 \\
&\qquad\quad \vdots \\
&\qquad a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \leq b_m \\
&\qquad x_1, x_2, \ldots, x_n \geq 0
\end{aligned}$$

For a **minimization** problem, the structure is the same but the objective is *Minimize* Z and constraints may have $\geq$ signs.

Step-by-Step Method (How I Formulate from a Word Problem)

I follow these four steps every time:

Step 1. Find decision variables. Ask: What do I need to decide? Give each a letter (usually x, y).

Step 2. Find objective function. Look for keywords: *profit, revenue, maximize* OR *cost, time, minimize*.

Step 3. Find constraints. Look for: *at most, no more than* ($\leq$) OR *at least, minimum* ($\geq$).

Step 4. Add non-negativity. Write $x \geq 0, y \geq 0$ (unless stated otherwise).

Example 1: Maximization (Factory Problem)

Problem in words: A factory makes A and B. Each A gives 40 profit, needs 2 machine hours and 1 labor hour. Each B gives 30 profit, needs 1 machine hour and 2 labor hours. Available: 100 machine hours, 80 labor hours. Maximize profit.
My solution (as I write in my notebook):

- Let $x =$ number of units of A.
- Let $y =$ number of units of B.
- Profit = $40x + 30y$. So Maximize $Z = 40x + 30y$.
- Machine hours: $2x + 1y \leq 100$.
- Labor hours: $1x + 2y \leq 80$.
- Non-negativity: $x \geq 0, y \geq 0$.

$$\begin{aligned} \text{Max } Z &= 40x + 30y \\ \text{s.t. } 2x + y &\leq 100 \\ x + 2y &\leq 80 \\ x, y &\geq 0 \end{aligned}$$

Example 2: Minimization (Diet Problem)

Problem in words: Need at least 24 protein and 18 carbs. Food X: 4 protein, 2 carbs, 5/unit. Food Y: 2 protein, 3 carbs, 3/unit. Minimize cost.

My solution:

- Let $x =$ units of food X.
- Let $y =$ units of food Y.
- Cost = $5x + 3y$. So Minimize $Z = 5x + 3y$.
- Protein: $4x + 2y \geq 24$.
- Carbs: $2x + 3y \geq 18$.
- Non-negativity: $x \geq 0, y \geq 0$.

$$\begin{aligned} \text{Min } Z &= 5x + 3y \\ \text{s.t. } 4x + 2y &\geq 24 \\ 2x + 3y &\geq 18 \\ x, y &\geq 0 \end{aligned}$$

Common Mistakes (I have made these before!)

- **Forgetting non-negativity:** Always write $x \geq 0, y \geq 0$.
- **Wrong inequality sign:** *At most* = $\leq$; *at least* = $\geq$.
- **Mixing coefficients:** Check which number goes with which resource.
- **Using = instead of $\leq$ or $\geq$:** Real problems almost never have exact equality.

Quick Reference Table (Keep this in mind)

Phrase in problem	Use this symbol
Maximize profit / revenue	Max Z
Minimize cost / time	Min Z
At most / no more than	$\leq$
At least / no less than	$\geq$
Exactly / equal to	$=$

Practice Problems (Do in notebook)

Q1. A carpenter makes chairs and tables. Chair: 200 profit, 3 wood, 2 labor. Table: 500 profit, 5 wood, 4 labor. Available: 150 wood, 100 labor. Maximize profit.

Q2. A farmer needs at least 20 tons fertilizer and 15 tons pesticide. Brand P: 2 fertilizer, 1 pesticide, 1000/bag. Brand Q: 1 fertilizer, 2 pesticide, 800/bag. Minimize cost.

Exit Check (Ask yourself before next class)

- Did I define all variables clearly?
- Is the objective function linear?
- Are all constraints linear?
- Did I include $x \geq 0, y \geq 0$?
- Can I solve a word problem alone now?

Next topic: Graphical Solution (using these LPPs).